Name: $\qquad$
Instructor: $\qquad$
Math 10170, Practice Exam 2
April 8, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

| PLEASE MARK YOUR ANSWERS WITH AN X, not a circle! |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1. (a) | (b) | (c) | (d) | (e) |
| 2. (a) | (b) | (c) | (d) | (e) |
| 3. (a) | (b) | (c) | (d) | (e) |
| 4. (a) | (b) | (c) | (d) | (e) |
| 5. (a) | (b) | (c) | (d) | (e) |
| 6. (a) | (b) | (c) | (d) | (e) |
| 7. (a) | (b) | (c) | (d) | (e) |


| Please do NOT write in this box. |  |
| ---: | :--- |
| Multiple Choice__ |  |
| 8. |  |
| 9. |  |
| 10. |  |
| 11. |  |
| Total | $\square$ |

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## Multiple Choice

1. ( 6 pts .) The probability distribution of a random variable $X$ given below, find $E(X)$ and the standard deviation $\sigma(X)$.

| k | $\operatorname{Pr}(\mathrm{X}=\mathrm{k})$ |
| :---: | :---: |
| -1 | $\frac{1}{2}$ |
| 0 | $\frac{1}{8}$ |
| 1 | $\frac{1}{16}$ |
| 2 | $\frac{1}{16}$ |
| 3 | $\frac{1}{4}$ |

(a) $-\frac{1}{4}$
(b) $\frac{1}{16}$
(c) $\frac{7}{16}$
(d) $\frac{1}{4}$
(e) $-\frac{3}{8}$
2.( 6 pts.) A basketball player with career field goal percentage of $80 \%$ takes 100 shots in a row. If his probability of making a basket on every shot is 0.8 , what is the longest run of misses that you would expect in the data?
(a) 13
(b) 3
(c) 10
(d) 6
(e) 20

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3. ( 6 pts.) Given a sequence with two values, success ( S ) and failure (F), with $N_{s}$ success' and $N_{f}$ failures, let $X$ denote the number of runs (of both S's and F's). Wald and Wolfowitz determined that for a random sequence of length N with $N_{s}$ success' and $N_{f}$ failures (note that $N=N_{s}+N_{f}$ ), the number of runs has mean and standard deviation given by

$$
E(X)=\mu=\frac{2 N_{s} N_{f}}{N}+1, \quad \sigma(X)=\sqrt{\frac{(\mu-1)(\mu-2)}{N-1}} .
$$

The distribution of $X$ is approximately normal if $N_{s}$ and $N_{f}$ are both bigger than 10 .
What is the value of $X$ for the following sequence of success' and failures and what is its Z-score (the number of standard deviations from the mean)?

$$
S F S S S F F S S S S S F F F F S S F S
$$

(a) $X=9, Z \approx 0.77$
(b) $X=5, Z \approx 2.69$
(c) $X=5, Z \approx-2.69$
(d) $X=9, Z \approx-0.77$
(e) $X=10, Z \approx 0.29$
4. ( 6 pts .) Statistics collected over a five year period for performance of wide receivers in the vertical jump at the NFL combine showed an average of 35.1 inches and a standard deviation of 3.2. Assuming that the distribution of the observations was mound shaped (roughly normally distributed), roughly what percentage of the wide receivers had a vertical jump height between 31.9 inches and 38.3 inches.
(a) $99.7 \%$
(b) $68 \%$
(c) $95 \%$
(d) $50 \%$
(e) $80 \%$

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5. ( 6 pts.) The following statistics on passing for quarterback Peyton Manning show his completion record for the 2014 season. $(\mathrm{CMP}=$ completed, $\mathrm{INC}=$ Incomplete)

|  | CMP | INC | Total |
| :---: | :---: | :---: | :---: |
| Home Games | 184 | 78 | 262 |
| Away Games | 211 | 124 | 335 |

If we choose a pass at random from the records, let $H$ be the event that it was in a home game and let $C$ be the event that it was complete, Which of the following statements are true?
(a) $\quad P(C \mid H)=\frac{184}{262}$
(b) $\quad P(H \mid C)=\frac{184}{597}$
(c) $\quad P(C \mid H)=\frac{262}{395}$
(d) $\quad P(C \mid H)=\frac{345}{397}$
(e) $\quad P(H \mid C)=\frac{262}{395}$

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6. ( 6 pts .) In a simplified model of a tennis serve, the server must decide whether to serve to the receiver's forehand (F) or to the backhand (B). The receiver must anticipate whether the serve will come to the forehand (F) or the backhand (B). For players $\operatorname{Robert}(R)$ and $\operatorname{Carl}(C)$, it is estimated that

- if Robert serves to the forehand (F) and Carl anticipates correctly, then there is a $50 \%$ chance that Robert will win the serve.
- On the other hand if Carl does not correctly anticipate the serve to the forehand, there is a $70 \%$ chance that Robert will win the serve.
- if Robert serves to the backhand (B) and Carl anticipates correctly, then there is a $40 \%$ chance that Robert will win the serve.
- On the other hand if Carl does not correctly anticipate the serve to the backhand, there is a $60 \%$ chance that Robert will win the serve.
Which of the following shows the correct payoff matrix for Robert for this constant sum game?
(a)

|  |  | Carl |  |
| :---: | :---: | :---: | :---: |
|  |  | F | B |
| Robert | F | 70 | 50 |
|  | B | 60 | 40 |

(b)

|  |  | Carl |  |
| :---: | :---: | :---: | :---: |
|  |  | F | B |
| Robert | F | 50 | 70 |
|  | B | 60 | 40 |

(c)

|  |  | Carl |  |
| :---: | :---: | :---: | :---: |
|  |  | F | B |
| Robert | F | 50 | 40 |
|  | B | 70 | 60 |

(e)

(d)

|  |  | Carl |  |
| :---: | :---: | :---: | :---: |
|  |  | F | B |
| Robert | F | 40 | 60 |
|  | B | 50 | 70 |

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7.( 6 pts .) The following graphs show the probability distributions of points scored per game played for Michael Jordan and Scottie Pippen.


Which of the following statements is true?
(a) The proportion of games played by Michael Jordan in which he scored more than 30 points was greater than 0.5 .
(b) The proportion of games played by Michael Jordan in which he scored more than 30 points was approximately equal to 0.04 .
(c) The average number of points scored per game for both players was the same.
(d) $\begin{aligned} & \text { The proportion of games played by Scottie Pippen played in which he scored } \\ & \text { more than } 30 \text { points was greater than } 0.5 \text {. }\end{aligned}$
(e) $\begin{aligned} & \text { The proportion of games played by Scottie Pippen in } \\ & \text { less than } 30 \text { points was approximately equal to } 0.02 \text {. }\end{aligned}$

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## Partial Credit

You must show your work on the partial credit problems to receive credit!
8. (14 pts.) The following statistics on field goal attempts by LeBron James were collected from 52 games of Basketball played this season:

|  | FGM | FGNM | Total = FGA |
| :---: | :---: | :---: | :---: |
| Home Games | 233 | 151 | 384 |
| Away Games | 283 | 227 | 510 |

Here
FGM means Field Goals Made;
FGNM means Field Goals attempted but not Made;
FGA means Field Goals Attempted.
(a) Use these statistics to estimate the probability that LeBron James will make the next field goal he attempts given that it is a home game.
(b) Use your estimate from part (a) to estimate the probability that LeBron James will make his first three attempts at field goal in his next home game, given that all of his shots at field goal are independent of each other.
(c) Are the events "Making a Field Goal" and "Playing in a Home Game" Independent? (Justify your answer)

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9.(12 pts.) On a reality TV show

- A contestant must complete a 2 step obstacle course to get to the next round.
- At the beginning of each stage of the obstacle course, the judge flips a fair coin to decide which task the contestant should complete.
- In the first stage of the obstacle course the contestant must either eat a bag of live worms or a bag of habanero peppers.
- If the contestant completes stage 1 of the obstacle course, they get to attempt stage two, otherwise, they are out of the competition.
- In the second stage of the obstacle course the contestant must either retrieve a stick from a tank of alligators or a ball from a pit of snakes.
- If the contestant completes stage 1 of the obstacle course, they get to move on to round two, otherwise, they are out of the competition.
Sydney has a $50 \%$ chance of finishing a bag of worms and a $20 \%$ chance of finishing a bag of habaneros. She has a $30 \%$ chance of retrieving a stick from a tank of alligators(and living on to tell the tale) and a $20 \%$ chance of getting a ball out of a pit of snakes (without being poisoned).

Use a tree diagram to determine the probability that Sydney will make it to the next round of the competition.

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10.(12 pts.) The following shows data for 50 consecutive passes for quarterback Drew Brees show whether each pass was complete(C) or incomplete (I):

C C C C I C C C I C I I C I C C C C C C C C C C I I I C C I C C C I C I I C C C I C C C C C C C C C
(a) How many runs (of C's and I's) are there in the data?
(b) If $X$ denotes the number of runs in a randomly generated sequence of C's and I's of length $N$ with $N_{C}$ C's and $N_{I}$ I's, $X$ has an approximately normal distribution with

$$
E(X)=\frac{2 N_{C} N_{I}}{N}+1, \quad \text { and } \quad \sigma(X)=\sqrt{\frac{(\mu-1)(\mu-2)}{N-1}} .
$$

Applying this distribution to the case given above, what is the $Z$ score of the above observed set of data?
(c) Using your knowledge of the empirical rule, would you say that the above data was randomly generated or not? Justify your answer.

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11. (20 pts.) Take Home Part

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| 2. (a) | ( $)$ | (c) | (d) | (e) |
| 3. (a) | (b) | (c) | ( $)$ | (e) |
| 4. (a) | ( $)$ | (c) | (d) | (e) |
| 5. ( $)$ | (b) | (c) | (d) | (e) |
| 6. (a) | (-) | (c) | (d) | (e) |
| 7. ( ) | (b) | (c) | (d) | (e) |


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